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Thesis

# AN ANALYSIS OF A PHASE SHIFT OSCILLATOR

by

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# INTRODUCTION

Audio frequency oscillators have become an indispensable part of the equipment necessary to one who repairs, designs or uses electronic instruments in industry, scientific and educational laboratories or in the radio service shop. The uses of an audio oscillator are many and varied, ranging from the location of rattles in cabinets and speaker cones to fidelity and distortion measurements in electrical equipment. There are a variety of commercial oscillators on the market today. The usual audio oscillators are of the feedback type, heterodyne type or RC type.

A satisfactory oscillator should meet the following requirements:

(1) Good frequency stability

(2) Low percentage distortion

(3) Constant power output

Any amplifying device is capable of generating oscillations if a sufficient portion of the output energy is fed back into the input in the proper phase so as to reinforce the input energy. If then part of the amplified power of a vacuum tube is fed back from the anode to the grid, by any one of several types of coupling devices, and is given the proper phase relation with respect to the anode, continued amplification sufficient to overcome the circuit losses produces sustained oscillations. Such a device is called a vacuum tube oscillator. The advantages of a vacuum tube oscillator over the alternator types are[1]:

(1) Wide frequency range

(2) Freedom from certain types of harmonics and richness in others

(3) Frequency stability

(4) Ease of frequency variation

(5) Portability and low cost

The production of oscillations in circuits without inductances is well known. Much work has been done on the glow-discharge, multivibrator, van der Pol and saw tooth oscillators; however, the production of sine waves from such circuits is a recent development. The above named oscillators are usually spoken of as relaxation oscillators. Relaxation oscillators are those in which one or more times in the cycle of oscillation one or more currents or voltages change abruptly. In these circuits, since there are no tuned circuits, the frequency of oscillation is determined primarily by the time constant of the grid resistance and capacitance. Relaxation oscillators are very unstable and are easily synchronized by a small external voltage. The output is highly distorted and harmonics as high as the 80th have been detected in the multivibrator type.[2]

The phase shift oscillator is a special type of resistance-capacitance tuned, sine wave oscillator that operates with a single tube. There are only a few references in the literature on sine wave-RC

1 H.J.Reich, Theory and Application of Electron Tubes, p-360

2 R.S.Glasgow, Principles of Radio Engineering, p-303

oscillators, and still fewer on the phase shift oscillator. Terman[1], Reich[2], and others in discussing this type of oscillator refer chiefly to an article by Ginzton, E. L., and Hollingsworth, L. M.[3] Kunde, W.W.[4] in a later article gave design and construction information on the oscillators discussed by Ginzton and Hollingsworth. The phase shift oscillator was patented by H. W. Nichols[5], Maplewood, New Jersey, assignor to Western Electric Co. The patent lists the following circuits which are of interest to the writer.

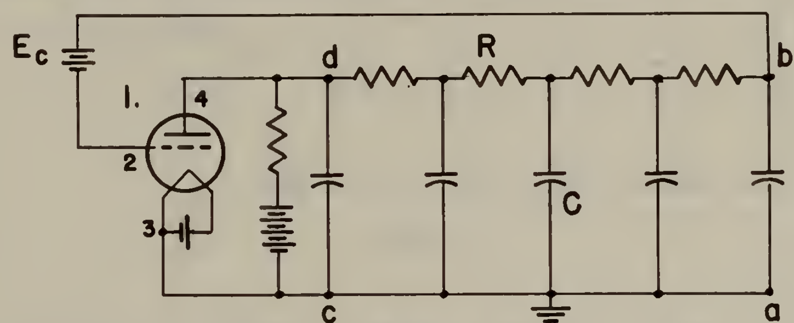



Fig. 1

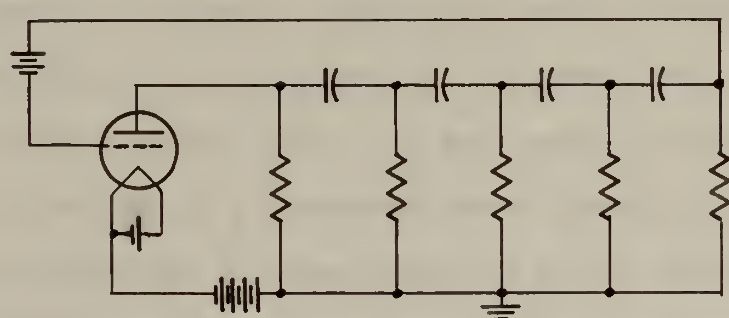


Fig. 2

1 Terman, Radio Engineering Handbook, p-506

2 Reich, Theory and Application of Electron Tubes, p-398

3 Ginzton and Hollingsworth, I.R.E.,Vol. 29, p-43, Feb. 1941

4 Kunde, Electronics, Nov. 1943,p-132

5 U.S.Patent 1,442,781, January 16, 1923 (filed July 7, 1921)

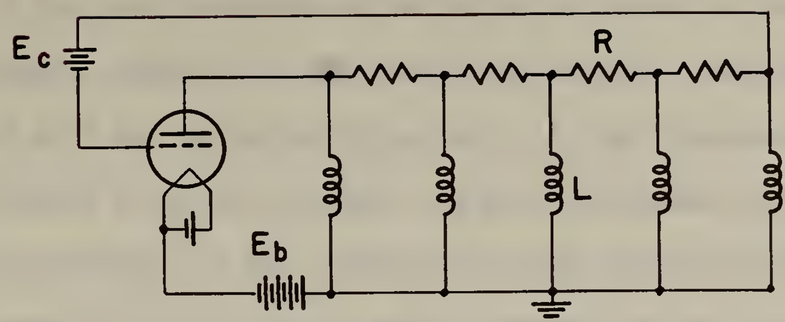



Fig. 3

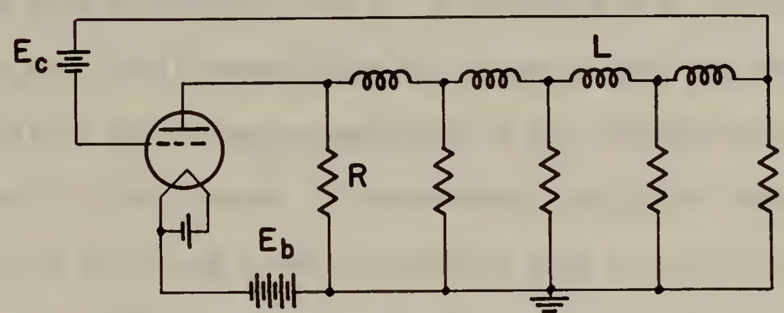



Fig. 4

The following description of the circuit is taken directly from a copy of the patent as recorded in the Boston Public Library. This is not a complete reprint of the patent, and the underlined sections are points to be noted that this writer has been unable to verify.

"Referring to Fig. 1, a thermionic three element amplifier 1, preferably highly evacuated, is shown associated with a network consisting of a plurality of meshes or sections, each having a series resistance R and a shunt capacity element C, and substantially free from inductance. The amplifier is provided with an impedance control element or grid 2, a hot filament cathode 3, and an anode or plate electrode 4. The space current for the amplifier is supplied by a source Eb in series with a very large resistance $R_1$. In the circuit of this figure and in those of each of the following figures a source Ec may be used to maintain

the grid at the most desirable potential with respect to the cathode."

"The input terminals of the amplifier, namely, those leading to the cathode and grid are connected to points a, b, the terminals of the end capacity element C of the network. The output terminals of the amplifier, namely, those leading to the cathode and plate are connected to points c,d, the terminals of the end capacity element C at the opposite end of the network from terminals a and b. A varying E.M.F. applied to the terminals a and b will accordingly be impressed upon the input circuit of the amplifier to produce an amplified E.M.F. across the output terminals c and d. The network in transferring amplified energy from points c and d to points a and b, operates both to shift its phase and to attenuate it. Each section of the network shifts the phase of the transmitted wave by a certain amount. If the phase shift throughout the entire network, that is, from points c and d to points a and b, is substantially 180 degrees or 180 degrees plus 360 degrees (sic), the amplified energy transmitted from terminals, c,d to terminals a,b will be in phase agreement with the applied E.M.F. thus giving rise to reamplification which occurs in consequence of the operation of amplifier 1, the amplified energy which reaches terminals a,b, will, after attenuation in the network be greater than the energy initially impressed at these terminals, and if the amplified energy is in phase agreement with the impressed energy, the amplifier and network may act an oscillator."

"Fig. 2 discloses an arrangement similar to Fig. 1, but including a network having shunt resistances R and series capacity elements C and substantially free from inductance. In this arrangement no separate

space current path is necessary for the source Eb."

"It is theoretically possible to design oscillators of each of the four types described with but two meshes in the feedback network. In general, it will be found desirable to use a greater number of meshes. For the arrangement of Figs. 1 and 3 the frequency of the oscillations produced, or for which the device will act as a reamplifying arrangement, decreases with an increasing number of section in the network. The higher frequency oscillations will therefore be obtained with but two section, and lower frequencies with larger number of sections. The frequency of the oscillations produced is a function of both the characteristics of the network and the voltage amplifying factor of the amplifier. For the circuit arrangements of Figs. 1 and 3, increasing the amplifying factor, decreases the frequency, the circuit being otherwise unvaried. The arrangements of Figs. 2 and 4 produce their lowest frequency oscillations with two meshes in the feedback network. As the number of meshes is increased, the frequency of oscillations produced by these arrangement increases. With a given network, the oscillators of these two figures produce increasing frequency oscillations as the voltage amplifying power of their amplifiers is increased and vice versa."

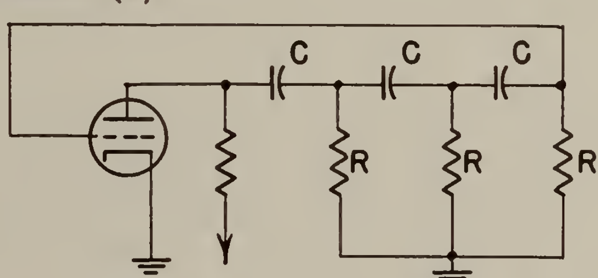
The above reprint is given in hopes that the reader, by means of comparison, will have a more comprehensive picture of the phase shift oscillator, - what it is and how it works.

## METHOD OF ANALYSIS

A given type of oscillator can usually be analyzed in several ways. The two common types of analysis are, (1) feedback and (2) negative resistance. The analysis to be used is on the basis of feedback with a comment on negative resistance.

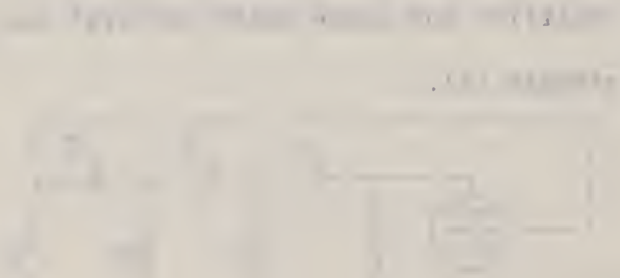
Since any amplifying system will oscillate if enough of its output is fed back into its input and in the proper phase to overcome circuit losses, this is equivalent to saying that a negative resistance has been shunted across its input. Negative resistance isone having characteristics opposite to real or positive resistance, i.e., in a circuit with negative resistance, a decreasing voltage gives an increasing current. In a vacuum tube circuit, if the negative resistance cancels the positive resistance of the circuit and the net circuit resistance is negative, and since negative resistance does not consume power, but furnishes its own power, the circuit acts as a generator. The energy required comes from the plate voltage supply.

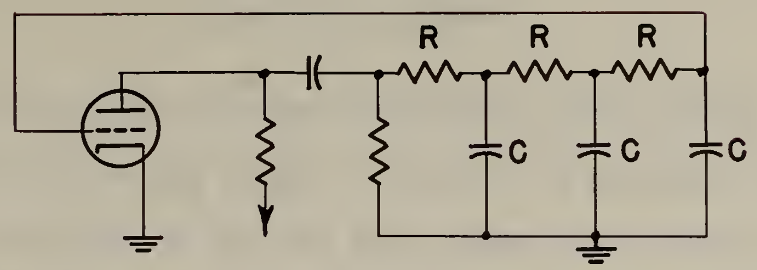
Ginzton and Hollingsworth[1] in their article on the phase shift oscillator listed the following basic circuits as "typical one tube resistance-capacitance coupled amplifier and phase shift network" and gave a mathematical analysis of circuit (a).



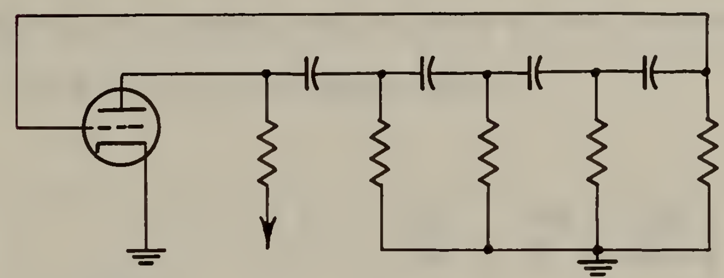

(a)

1 Ginzton,E.L., Hollingsworth, L.M.,Proc. I.R.E. Feb. 1941 p-43

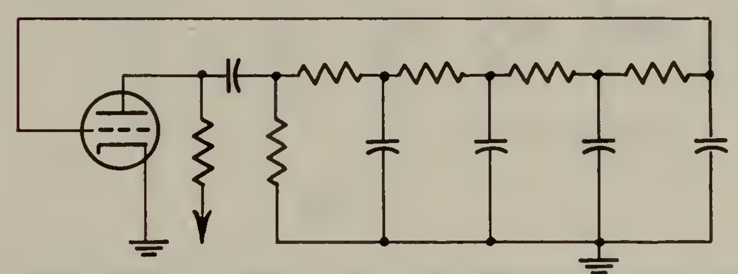





(b)



(c)



(d)

It is the desire of this writer to give a mathematical analysis of a circuit similar to (c) and give the results of laboratory tests made on the circuit.

THEORETICAL ANALYSIS

A three-or-more-mesh resistance-capacitance phase shifting network may be connected between the output and input of an amplifier tube with the circuit so proportioned that the total phase shift between the plate and grid terminals is 180 degrees out of phase at the frequency of oscillation desired. Such a circuit is shown below.

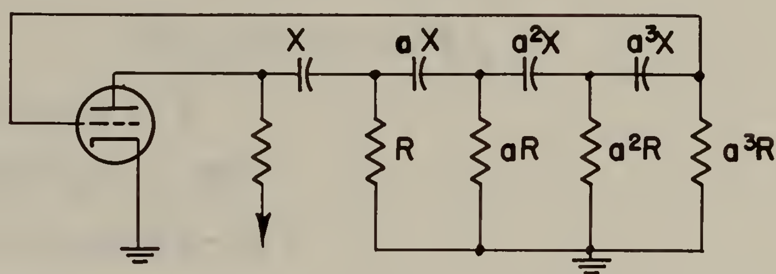



Fig. 5

The circuit analyzed by Ginzton and Hollingsworth uses meshes of constant impedance which results in the use of a tube which possesses an amplification equal to or greater than 29 for oscillations to start, thus a hi-mu triode or pentode is required. The use of four-mesh network where the mesh impedance varies by a constant ratio reduces the amount of amplification needed for oscillation and makes the selection of tubes less critical.

An equivalent circuit diagram of the four mesh network used is given in Fig. 6.

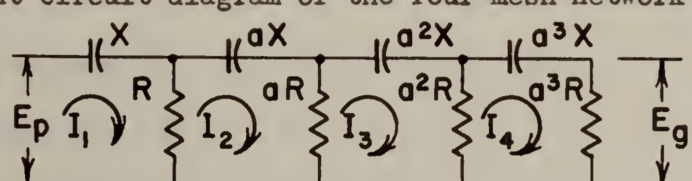



Fig. 6

a is the impedance transformation ratio between meshes. The circuit

will oscillate provided Eg times k, where k is the amplication of the circuit, gives the plate voltage, Ep required in the phase shift network. Using the conventional loop method gives the following set of equations:

$$(R-jX)I_1-RI_2 = Ep \tag{1}$$

$$-RI_1+R(1+a)-jaX)I_2-aRI_3 = 0 \tag{2}$$

$$-aRI_2+\left[aR(1+a)-ja^2X\right]I_3-a^2RI_4 = 0 \tag{3}$$

$$-a^2RI_3+\left[a^2R(1+a)-ja^3X\right] I_4 = 0 \tag{4}$$

dividing each equation by R

$$(1-jX/R)I_1-I_2 = Ep/R \tag{1a}$$

$$-I_1+(1+a-jaX/R)I_2-aI_3 = 0 \tag{2a}$$

$$-\cancel{a}I_2+\cancel{a}(1+a-jaX/R)I_3-aI_4 = 0 \tag{3a}$$

$$-\cancel{a}^2I_3+\cancel{a}^2(1+a-jaX/R)I_4 = 0 \tag{4a}$$

Let $X/R = m$ and $(1+a-jam) = d$

$$(1-jm)I_1-I_2 = Ep/R$$

solving for $I_1$

$$I_1 = (I_2+Ep/R)\left[1/(1-jm)\right]$$

rationalizing

$$I_1 = (I_2+Ep/R)\left[(1+jm)/(1+m^2)\right]$$

Let $(1+jm)/(1+m^2) = b$

$$I_1 = bI_2+Epb/R \tag{5}$$

$$-I_1+dI_2-aI_3 = 0 \tag{6}$$

$$-I_2+dI_3-aI_4 = 0 \tag{7}$$

$$-I_3+dI_4 = 0 \tag{8}$$

Substitute (5) in (6)

$$-bI_2-Epb/R+dI_2-aI_3 = 0$$

$$(d-b)I_2-aI_3-Epb/R = 0 \qquad (9)$$

Substitute (8) in (9)

$$(d-b)I_2-adI_4-Epb/R = 0 \qquad (10)$$

Substitute (8) in (7)

$$-I_2+(d^2-a)I_4 = 0 \qquad (11)$$

Solving (10) and (11) for $I_4$

$$\left[(d-b)(d^2-a)-ad\right]I_4 = (b/R)Ep \qquad (12)$$

Eg is 180 degrees out of phase with Ep, therefore

$$Ep = -kEg \qquad \text{(k is the circuit amplification)}$$

From loop 4

$$Eg = a^2RI_4$$

$$Eg = \frac{a^3bEp}{d^3-bd^2-2ad+ab} = -(1/k)Ep$$

$$-ka^3b = d^3-bd^2-2ad+ab \qquad (13)$$

but $1/b = 1-jm$ multiplying (13) by $1/b$

$$-ka^3 = d^3(1-jm) - d^2-2ad(1-jm)+a \qquad (14)$$

Equation (14 shows that in order to satisfy the conditions for oscillation the amplification must be real negative number. Hence the imaginary part of the complex number on the right hand side of the

equation must be zero.

Since

$$d = (a+1)-jam$$

$$d^2 = (a+1)^2-a^2m^2-2jam(a+1)$$

$$d^3 = (a+1)^3-3(a+1)a^2m^2+j\left[a^3m^3-3am(a+1)^2\right]$$

substituting these values in (14) gives

$$-ka^3 = \left\{(a+1)-3(a+1)a^2m^2-j\left[a^3m^3-3am(a+1)^2\right]\right\}\left\{1-jm\right\} - \left[(a+1)^2-a^2m^2-2jam(a+1)\right]-2a(1-jm)(a+1-jam)+a \quad (15)$$

Equating the j-term to zero

$$a^3m^3-(a+1)^3m-3am(a+1)(a-am^2+1)+2am(a+1)+2a(2am+m) = 0$$

dividing by m

$$4a^3m^2-4a^3-3a^2-2a-3a^2m^2-1 = 0$$

$$4a^3+3a^2+2a+1 = (4a^3+3a^2)m^2$$

therefore

$$m = \sqrt{1+(2a+1)/(4a^3+3a^2)} \quad (16)$$

$$m = X/R$$

and since $X = 1/(2\pi fC)$

$$C = \frac{1}{(2\ Rf)\sqrt{1+(2a+1)/(4a^3+3a^2)}}$$

or

$$f = \frac{1}{(2\pi RC)\sqrt{1+(2a+1)/(4a^3+3a^2}} \quad (17)$$

The amplification of the circuit is now found by taking the real part of equation (15) and solving for k.

$$-ka^3 = a^3m^4+(a+1)^3-3(a+1)am^2(2a+1)+a^2m^2-(a+1)^2-2a^2+2a^2m^2-a$$

$$-ka^3 = a^3m^4-6a^3m^2+a^3-6a^2-3am^2$$

dividing by $a^3$

$$-ka = m^4-6m^2+1-6m^2/a-3m^2/a^2$$

$$k = 3m^2(2+2/a+1/a^2)-m^4-1 \qquad (1)$$

Equations (18) and (17) gives the frequency of oscillation and the amplification necessary in terms of the circuit constants.

If a, impedance transformation ratio between meshes, is equal to 1, then

$$m = \sqrt{1+3/7} = \sqrt{10/7}$$

Substituting this value in (18) gives a value of k equal to 18.4. This is the amplification needed if all the capacitances C, and resistances R, are alike, which may be compared with an amplification of 29 for a similar 3-mesh network.

If the impedance ratio has values larger than one the necessary amplification will decrease. If the impedance ratio is 2, then m will be equal to $\sqrt{49/44}$, and the necessary amplification 8.63. Table 1 gives other values for a, m, and k, and Fig. 7 shows how m and k vary with a.

It can be seen from the table that an impedance ratio greater than 2 gives only slight increase in circuit efficiency while a change from 1 to 2 decreases the needed amplification from 18.4 to 8.63. Blocking in the oscillator is most apt to occur when high values of grid leak resistance are use, hence the value of the a used is limited.

| a | m | k |
|---|---|---|
| 0.5 | 1.612 | 70.24 |
| 1.0 | 1.195 | 18.40 |
| 1.5 | 1.095 | 11.12 |
| 2.0 | 1.054 | 8.63 |
| 2.5 | 1.038 | 7.40 |
| 3.0 | 1.025 | 6.65 |
| 00 | 1.000 | 4.00 |

Table 1

In the 4-mesh network the phase shift is 45 degrees per section and m is equal to 1, the grid voltage, Eg is equal to $(\cos 45)^4$Ep. This gives k a value of 4, if then,as <u>a</u> becomes very large, the amplification (k) necessary for any number of meshes can be written as

$$k = \frac{1}{(\cos 180/n)^n}$$

Table 2 and Fig. 8 show the amplification for values of n.

| No. of meshes n | Amplification needed k |
|---|---|
| 2 | infinite |
| 3 | 8.000 |
| 4 | 4.000 |
| 5 | 2.850 |
| 6 | 2.137 |
| infinite | 1.000 |

Table 2

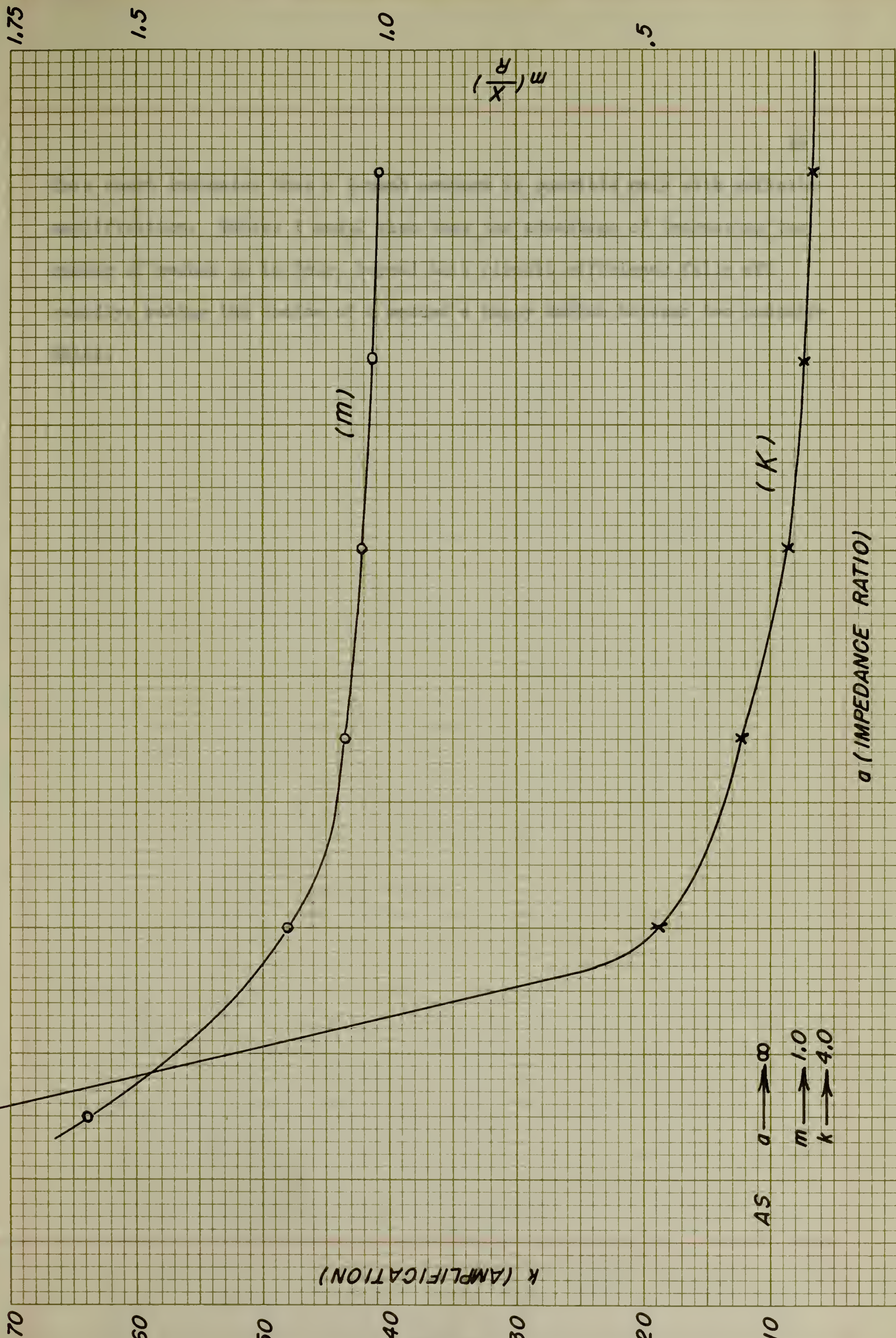

k (AMPLIFICATION)
70
60
50
40
30
20
10
a (IMPEDANCE RATIO)
.5
1
1.5
2
2.5
3
m (X/R)
1.75
1.5
1.0
.5
(m)
(K)
AS a → ∞
m → 1.0
k → 4.0

Fig. 7

This chart indicates that a 2-mesh network is possible only with infinite amplification. Tables 1 and 2 also show the advantage of increasing the number of meshes up to four, beyond this circuit efficiency falls off rapidly, making the choice of 4 meshes a happy medium between two undesirables.

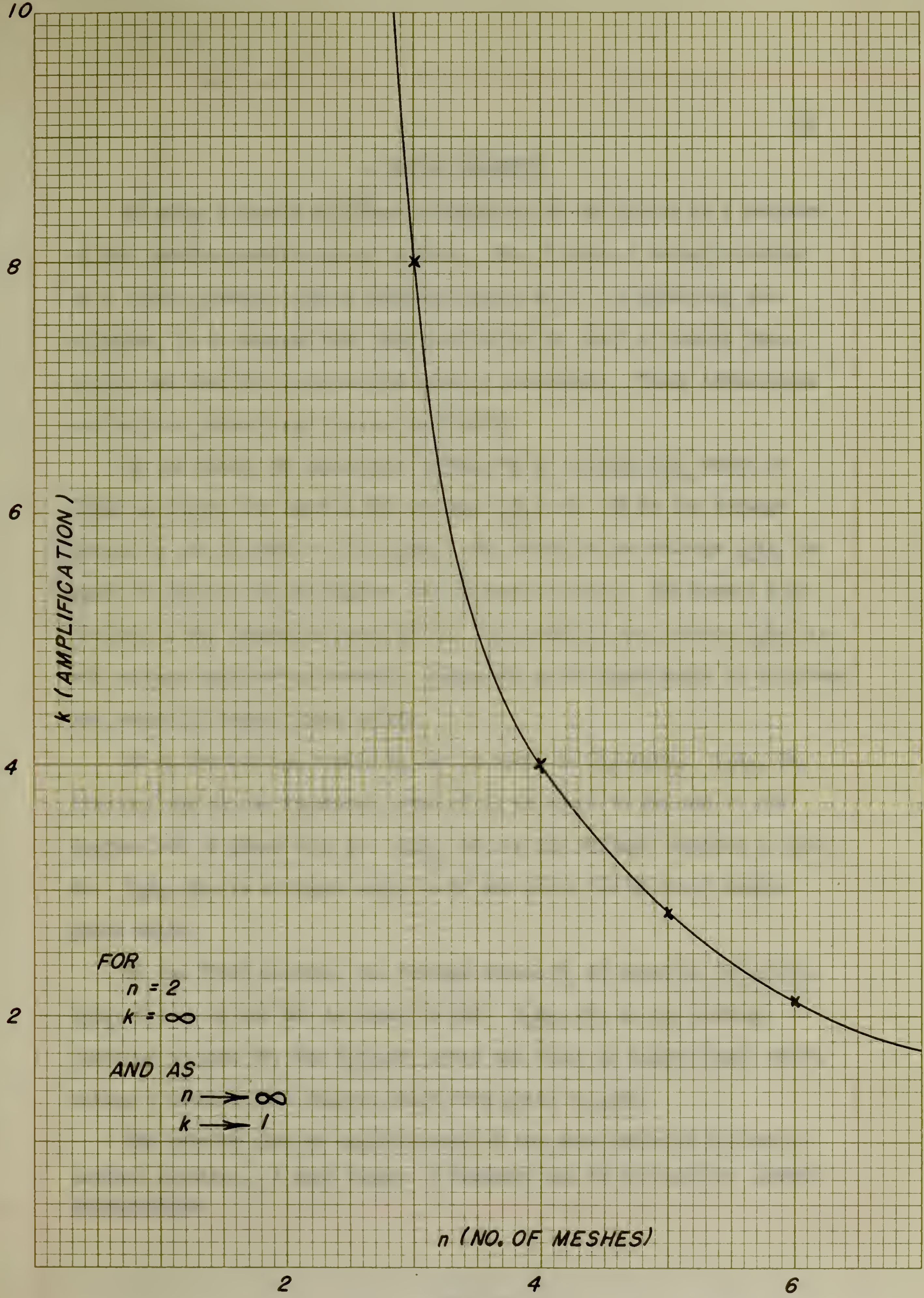



Fig. 8

## A VECTOR SOLUTION

By using slightly different assumptions we may arrive at a solution of the problem graphically by vectors. Fig. 9 gives a vector solution of a 4-mesh network with an impedance ratio of 2. In obtaining this solution, it is assumed that each mesh gives an exact 45 degree phase shift, and that the capacitances have no resistance. These assumptions can be made without any loss of generality.

On the graph, OA represents voltage Eg or voltage $I_4R_4$ which is taken as unity, and leading the voltage $I_4X_4$, AB. OB is the voltage across $R_3$ and is equal to $(I_3-I_4)R_3$. Oa, which is the voltage $I_4R_3$, is equal to $\frac{1}{2}$OA and is 180 degrees out of phase with it. (Oa equals $\frac{1}{2}$ OA because of the impedance ratio of 2). aB, which is the voltage $I_3R_3$, is the voltage required to give OB. $I_3X_3$, BC, is at right angle to aB gives the second 45 degree phase shift.

OC is the voltage across $R_2$ and is equal to $(I_2-I3)R_2$. $I_3R_3$, Ob, like Oa, due to the impedance ratio of 2, is equal to $\frac{1}{2}$aB and is 180 degrees out of phase with it. $I_2R_2$, bC, is the voltage required to give OC. $I_2X_2$, CD, is at right angle to bC and gives the third 45 degree phase shift.

In the first section, the voltage across $R_1$ is equal to $(L_1-I_2)R_1$. $I_2R_1$, Oc, as Ob and Oa, is equal to $\frac{1}{2}$bC. $I_1R_1$, cD, is the voltage required to give OD, and $I_1X_1$,DE, gives the final 45 degree phase shift, making a total of 180 degrees shift from plate to grid.

The results give an amplification of the same order as the mathematical solution. A high degree of accuracy can be obtained by careful construction.

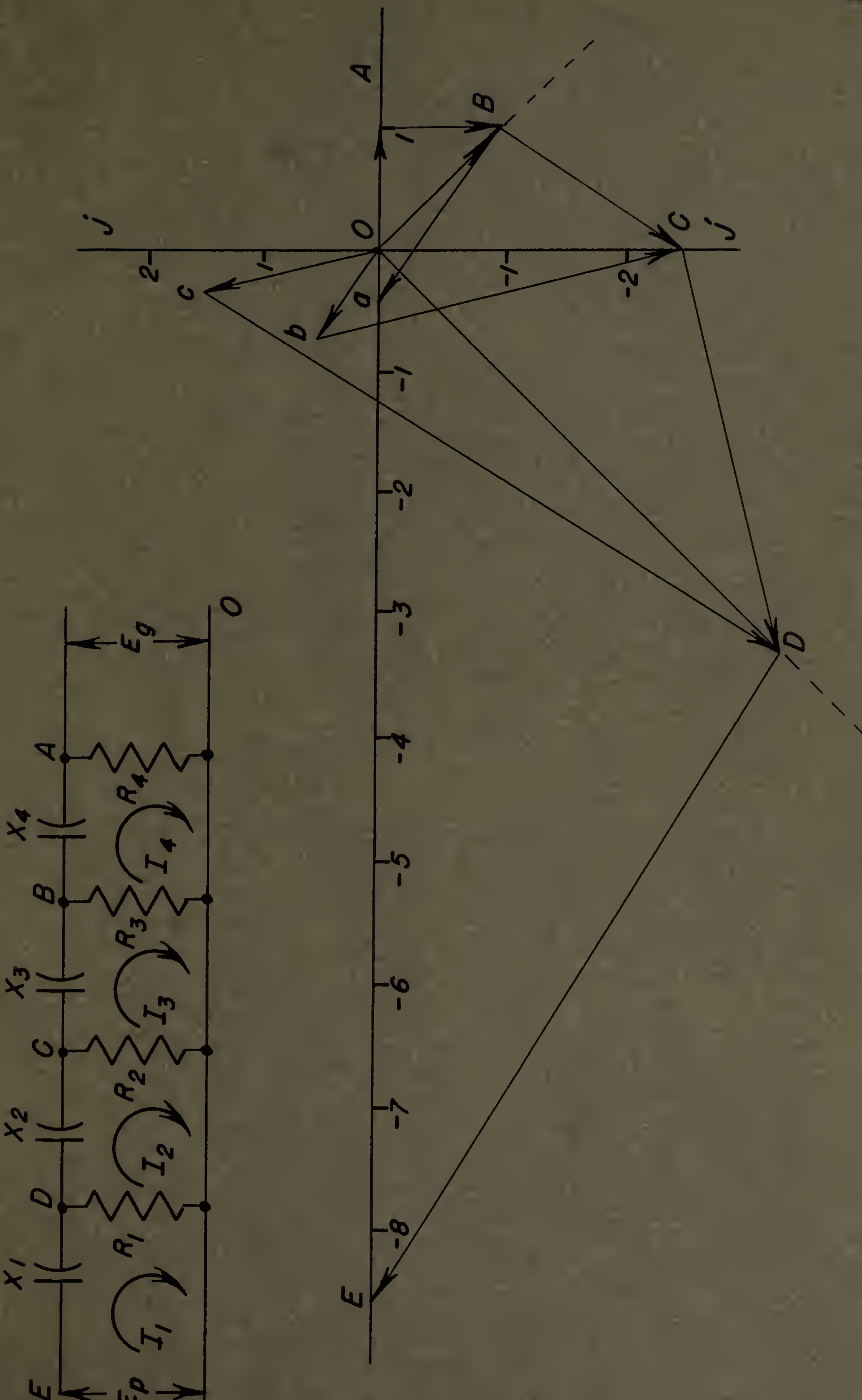



Fig. 9

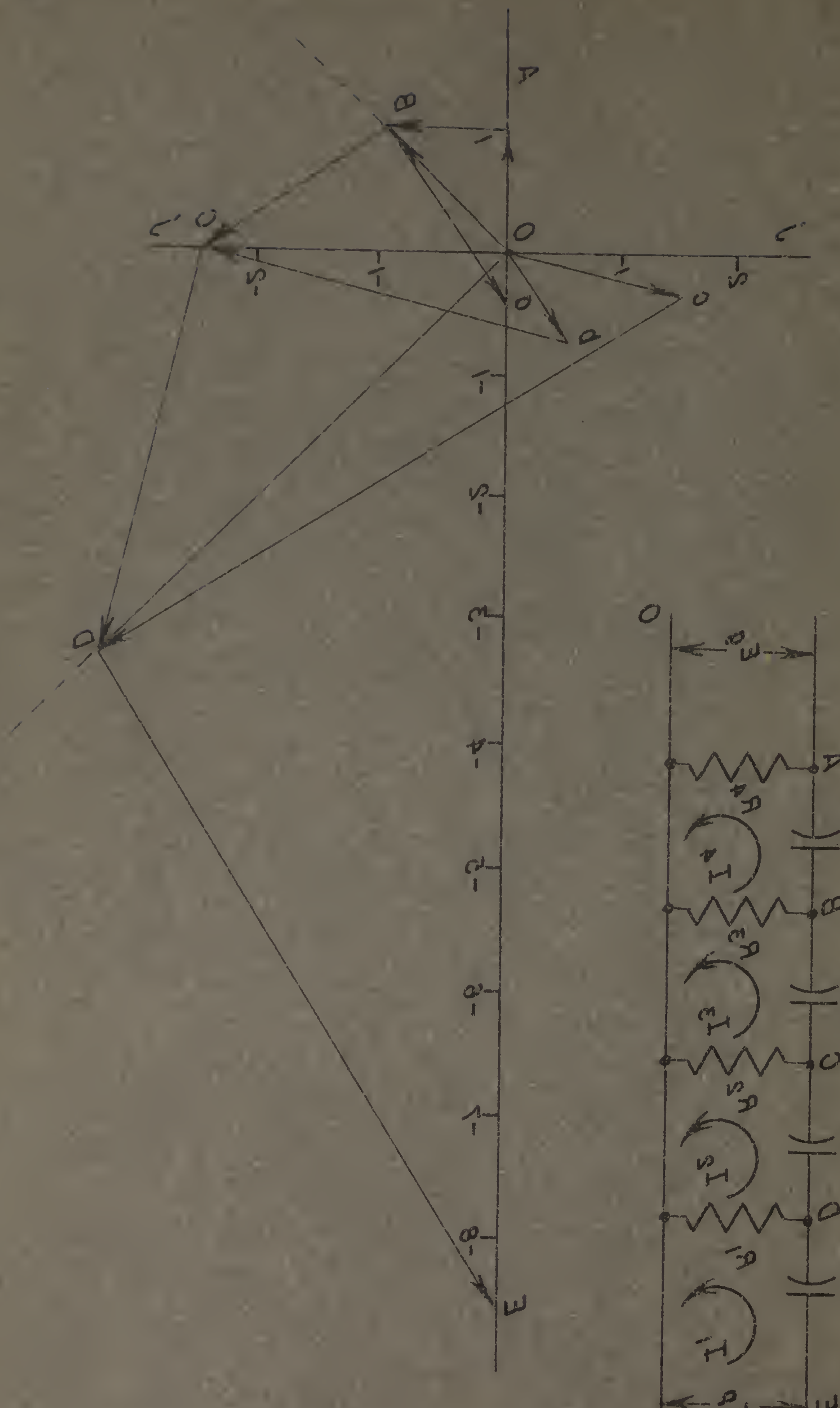


Fig. 4

FINDINGS AND CONCLUSIONS

One of the oscillator circuits designed and checked in the laboratory is shown in Fig. 10. $V_1$, 6C4, a low-mu triode, is the oscillator, and $V_2$,6C4, is used as a direct coupled cathode follower. By direct coupling to the cathode follower large power amplification is obtained with no appreciable phase shift. $V_2$ acts as an impedance transformer furnishing a constant voltage to the phase shift network, and a source of power to the output with little danger of loading the oscillator. $V_2$ also helps isolate the phase shift network from the dynamic characteristics of the oscillator tube.

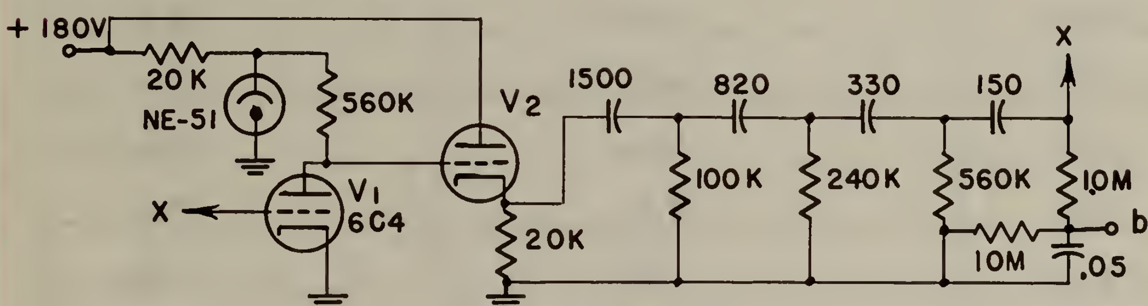



Fig. 10

By applying a fixed bias to $V_1$ as shown in Fig. 11.

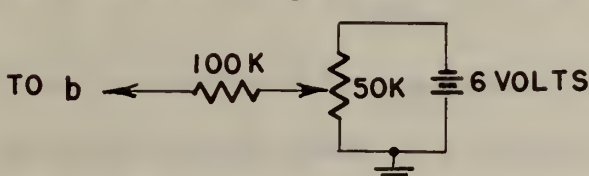



Fig. 11

it is possible to determine how the output voltage varies with the bias and at the same time get an idea as to the efficiency of the circuit. Table 1 shows how the output voltage varied with the bias. It may be noted from the table with an 80 volt plate supply to the oscillator

a maximum of 45.2 a.c volts (peak to peak) output is obtained.

Oscillator plate voltage - 80 volts

| Fixed bias at b | Output at a | |
|---|---|---|
| | a.c. volts (peak to peak) | d.c. volts |
| -1.00 | 1.8 | 23.6 |
| -1.25 | 12.7 | 28.0 |
| -1.50 | 20.3 | 31.5 |
| -1.75 | 28.3 | 34.0 |
| -2.00 | 34.4 | 36.0 |
| -2.20 | 38.1 | 38.5 |
| -2.50 | 42.9 | 42.0 |
| -2.75 | 45.2 | 43.0 |
| -2.80 | 44.8 | 43.0 |
| -2.90 | 43.7 | 43.0 |
| -3.00 | 36.7 | 44.2 |
| -3.10 | 15.8 | 44.5 |

Table 3

Fixed bias is rarely used in a practical oscillator. Letting the oscillator run free the grid picks up electrons and biases itself automatically with amplitude. This self-regulating feature of the oscillator is the chief factor in its good frequency stability and low harmonic output.

Frequency instability results from, (1) mechanical vibration of circuit elements, (2) temperature changes, (3) variations in loading, and (4) operating voltage variations. Table 4 shows changes in frequency

which results from changes in plate supply voltage.

| Plate voltage | A.C. output R.M.S. | Frequency in cycles |
|---|---|---|
| 180.0 | 37.0 | 930 |
| 157.5 | 31.7 | 930 |
| 135.0 | 26.3 | 935 |
| 112.5 | 21.0 | 940 |
| 90.0 | 15.5 | 945 |
| 67.5 | 9.0 | 950 |

Table 4

A change of 167% change in plate voltage produces a change of 2.1% change in frequency.

DESIGN FEATURES

Once the value, $m = X/R$, has been determined from equation (16) or taken from Table 1, the capacitance for a particular frequency can be determined

$$X/R = \frac{1}{2\pi fCR} \qquad \text{or}$$

$$C = \frac{10^{12}}{2\pi fmR} \ \mu\mu\text{fd.} \quad (\text{f in cycles})$$

The values of the capacitance and resistance found by using the formula may not give the desired frequency due to manufacturer's tolerances, but decreasing the resistance or capacitance in any section will raise the frequency and vice versa.

## SUMMARY

The phase shift oscillator described in this treatise is a single tube, RC oscillator with sine wave output. The phase shift network required to give an in-phase voltage at the grid of a tube from its plate terminal may consist of three or more phase shifting sections; a two-section network would require infinite amplification in the oscillator circuit. The phase shift oscillator will work at frequencies as high as fifty thousand cycles, but it is usually designed to operate at the medium or low audio frequencies.

This type of oscillator was patented by H.W.Nichols in 1923. A part of the patent is reprinted, hoping, by comparison, the reader will get a more comprehensive picture of how the oscillator works.

The oscillator with a four-mesh network is analyzed mathematically and graphically. The phase shift network used in other descriptions of this type of oscillator use common capacitances and resistances; in this analysis, an impedance transformation ratio, a, is used in order to decrease the amount of amplification needed for the circuit to oscillate. It is shown that as the impedance ratio, a, increases the amplification needed for oscillation decreases, and when a approaches infinity the amplification needed approaches four. Also the amplification is shown to be a function of the number of meshes used, as the number of meshes increases the amplification decreases.

The frequency at which the oscillator will work is found to be a function of the constants of the circuit, i.e., the frequency is inversely proportional to the values of R, C and a; a equals the ratio of Xc to R.

A simple vector solution is given; the results indicate an amplification of the same order as found in the rigorous mathematical solution.

The results obtained from a laboratory model of the oscillator verify the mathematical solution. The model has been incorporated in a proposed experiment and the results have been far above expectations.

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